N13/5/MATME/SP2/ENG/TZ0/XX/M

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MARKSCHEME

November 2013

MATHEMATICS

Standard Level

Paper 2

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning.**

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document "**Mathematics SL: Guidance for e-marking November 2013**". It is **essential** that you read this document before you start marking. In particular, please note the following. Marks must be recorded using the annotation stamps, using the new scoris tool. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks. Do **not** use the ticks with numbers for anything else.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, all the working **must** have annotations stamped to show what marks are awarded. This includes any zero marks.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M, A, R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate)
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted, neither should ***N*** marks be awarded for these answers.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

*Candidates should NO LONGER be penalized for an accuracy error **(AP)**. Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant **A** marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for **FT**. Further information on which answers are accepted is given in a separate booklet, along with examples. It is **essential** that you read this carefully.*

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*eg* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

Clarification of intermediate values accuracy instructions

Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer. However, do not penalise inaccurate intermediate values that lead to an acceptable final answer.

11 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation
The Mathematics SL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

12 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

13 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

14. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

SECTION A

1. (a) $A^{-1} = \begin{pmatrix} -1 & 4 & -1.5 \\ 1 & -4 & 2 \\ -1 & 5 & -2.5 \end{pmatrix}$ ***A2*** ***N2***

Note: Award ***A1*** for 6, 7 or 8 correct elements.

[2 marks]

(b) attempt to solve equation ***(M1)***

eg multiplying by A^{-1}, setting up system of equations

$X = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ (accept $x = 2,\ y = 3,\ z = -1$) ***A2*** ***N3***

[3 marks]

Total [5 marks]

2. (a) valid approach ***(M1)***

eg $f(x) = 0$, sketch of parabola showing two x-intercepts

$x = 1,\ x = 4$ $\left(\text{accept } (1,\ 0),\ (4,\ 0)\right)$ ***A1A1*** ***N3***

[3 marks]

(b) attempt to substitute either limits or the function into formula involving f^2 ***(M1)***

eg $\int_1^4 \left(f(x)\right)^2 \mathrm{d}x,\ \pi\int\left((x-1)(x-4)\right)^2$

$\text{volume} = 8.1\pi$ (exact), 25.4 ***A2*** ***N3***

[3 marks]

Total [6 marks]

3. (a) expressing f as $x^{\frac{4}{3}}$ *(M1)*

$f'(x) = \frac{4}{3}x^{\frac{1}{3}} \quad \left(= \frac{4}{3}\sqrt[3]{x}\right)$ ***A1*** ***N2***

[2 marks]

(b) attempt to integrate $\sqrt[3]{x^4}$ *(M1)*

eg $\frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1}$

$\int f(x)\mathrm{d}x = \frac{3}{7}x^{\frac{7}{3}} - \frac{x}{2} + c$ ***A1A1A1*** ***N4***

[4 marks]

Total [6 marks]

4. (a) correct approach *(A1)*

eg $0.5 = 0.2 + \mathrm{P}(B)$, $\mathrm{P}(A \cap B) = 0$

$\mathrm{P}(B) = 0.3$ ***A1*** ***N2***

[2 marks]

(b) Correct expression for $\mathrm{P}(A \cap B)$ (seen anywhere) ***A1***

eg $\mathrm{P}(A \cap B) = 0.2\mathrm{P}(B)$, $0.2x$

attempt to substitute into correct formula for $\mathrm{P}(A \cup B)$ *(M1)*

eg $\mathrm{P}(A \cup B) = 0.2 + \mathrm{P}(B) - \mathrm{P}(A \cap B)$, $\mathrm{P}(A \cup B) = 0.2 + x - 0.2x$

correct working *(A1)*

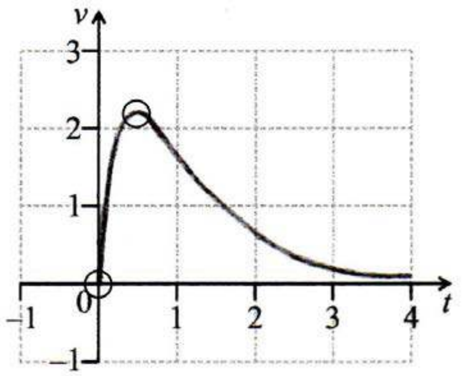
eg $0.5 = 0.2 + \mathrm{P}(B) - 0.2\mathrm{P}(B)$, $0.8x = 0.3$

$\mathrm{P}(B) = \frac{3}{8}$ $(= 0.375$, exact$)$ ***A1*** ***N3***

[4 marks]

Total [6 marks]

5. (a)



A1A2 *N3*

Notes: Award *A1* for approximately correct domain $0 \le t \le 4$.

The shape must be approximately correct, with maximum skewed left. **Only** if the shape is approximately correct, award *A2* for all the following approximately correct features, in circle of tolerance where drawn (accept seeing correct coordinates for the maximum, even if point outside circle):

Maximum point, passes through origin, asymptotic to *t*-axis (but must not touch the axis).

If only two of these features are correct, award ***A1.***

[3 marks]

(b) valid approach (including 0 and 3) ***(M1)***

eg $\int_0^3 10te^{-1.7t}\,dt$, $\int_0^3 f(x)$, area from 0 to 3 (may be shaded in diagram)

distance $= 3.33$ (m) ***A1*** ***N2***

[2 marks]

(c) recognizing acceleration is derivative of velocity ***(R1)***

eg $a = \frac{dv}{dt}$, attempt to find $\frac{dv}{dt}$, reference to maximum on the graph of v

valid approach to find v when $a = 0$ (may be seen on graph) ***(M1)***

eg $\frac{dv}{dt} = 0$, $10e^{-1.7t} - 17te^{-1.7t} = 0$, $t = 0.588$

velocity $= 2.16$ (m s^{-1}) ***A1*** ***N3***

Note: Award ***R1M1A0*** for $(0.588,\ 2.16)$ if velocity is not identified as final answer

[3 marks]

Total [8 marks]

6. **Note**: There may be slight differences in answers, depending on whether candidates use tables or GDCs, or their 3 sf answers in subsequent parts. Do not penalise answers that are consistent with **their** working and check carefully for ***FT***.

(a) attempt to standardize *(M1)*

eg $z = \frac{21.8-20}{1.25}$, 1.44

$\mathrm{P}(T < 21.8) = 0.925$ ***A1*** ***N2***

[2 marks]

(b) attempt to subtract probabilities *(M1)*

eg $\mathrm{P}(T < 21.8) - \mathrm{P}(T < k) = 0.3$, 0.925–0.3

$\mathrm{P}(T < k) = 0.625$ ***A1***

EITHER

finding the z-value for 0.625 *(A1)*

eg $z = 0.3186$ (from tables), $z = 0.3188$

attempt to set up equation using **their** z-value *(M1)*

eg $0.3186 = \frac{k-20}{1.25}$, $-0.524 \times 1.25 = k - 20$

$k = 20.4$ ***A1*** ***N3***

OR

$k = 20.4$ ***A3*** ***N3***

[5 marks]

Total [7 marks]

7. (a) (i) valid approach (may be seen on diagram) *(M1)*

eg Q to 6 is x

$\mathrm{PQ} = 6 - 2x$ ***A1*** ***N2***

(ii) $A = (6-2x)\sqrt{6x - x^2}$ ***A1*** ***N1***

[3 marks]

(b) (i) recognising $\frac{\mathrm{d}A}{\mathrm{d}x}$ at $x = 2$ needed (must be the derivative of area) *(M1)*

$\frac{\mathrm{d}A}{\mathrm{d}x} = -\frac{7\sqrt{2}}{2}$, -4.95 ***A1*** ***N2***

(ii) $a = 0.879$ $b = 3$ ***A1A1*** ***N2***

[4 marks]

Total [7 marks]

SECTION B

8.

Notes: In this question, there may be slight differences in answers, depending on which values candidates carry through in subsequent parts. Accept answers that are consistent with their working.
Candidates may have their GDCs in degree mode, leading to incorrect answers. If working shown, award marks in line with the markscheme, with ***FT*** as appropriate.
Ignore missing or incorrect units.

(a) evidence of choosing sine rule ***(M1)***

eg $\frac{\sin \hat{A}}{a} = \frac{\sin \hat{B}}{b}$

correct substitution ***(A1)***

eg $\frac{\sin \hat{A}}{10.4} = \frac{\sin 1.058}{12.2}$

$\text{B}\hat{\text{A}}\text{C} = 0.837$ ***A1 N2***

[3 marks]

(b) **METHOD 1**

evidence of subtracting angles from π ***(M1)***

eg $\text{A}\hat{\text{B}}\text{C} = \pi - A - C$

correct angle (seen anywhere) ***A1***

$\text{A}\hat{\text{B}}\text{C} = \pi - 1.058 - 0.837$, $1.246, 71.4^\circ$

attempt to substitute into cosine or sine rule ***(M1)***

correct substitution ***(A1)***

eg $12.2^2 + 10.4^2 - 2 \times 12.2 \times 10.4 \cos 71.4$, $\frac{\text{AC}}{\sin 1.246} = \frac{12.2}{\sin 1.058}$

$\text{AC} = 13.3$ (cm) ***A1 N3***

METHOD 2

evidence of choosing cosine rule ***M1***

eg $a^2 = b^2 + c^2 - 2bc \cos A$

correct substitution ***(A2)***

eg $12.2^2 = 10.4^2 + b^2 - 2 \times 10.4 b \cos 1.058$

$\text{AC} = 13.3$ (cm) ***A2 N3***

[5 marks]

continued …

Question 8 continued

(c) **METHOD 1**

valid approach *(M1)*

eg $\cos \hat{AOC} = \frac{OA^2 + OC^2 - AC^2}{2 \times OA \times OC}$, $\hat{AOC} = 2 \times \hat{ABC}$

correct working *(A1)*

eg $13.3^2 = 7^2 + 7^2 - 2 \times 7 \times 7 \cos \hat{AOC}$, $O = 2 \times 1.246$

$\hat{AOC} = 2.492 \quad (142.8^\circ)$ *(A1)*

EITHER

correct substitution for arc length (seen anywhere) ***A1***

eg $2.492 = \frac{l}{7}$, $l = 17.4$, $14\pi \times \frac{142.8}{360}$

subtracting arc from circumference *(M1)*

eg $2\pi r - l$, $14\pi - 17.4$

OR

attempt to find $\hat{AOC}$ reflex *(M1)*

eg $2\pi - 2.492$, 3.79, $360 - 142.8$

correct substitution for arc length (seen anywhere) ***A1***

eg $l = 7 \times 3.79$, $14\pi \times \frac{217.2}{360}$

THEN

arc $ABC = 26.5$ ***A1*** ***N4***

METHOD 2

valid approach to find $\hat{AOB}$ or $\hat{BOC}$ *(M1)*

eg choosing cos rule, twice angle at circumference

correct working for finding **one** value, $\hat{AOB}$ or $\hat{BOC}$ *(A1)*

eg $\cos \hat{AOB} = \frac{7^2 + 7^2 - 12.2^2}{2 \times 7 \times 7}$, $\hat{AOB} = 2.116$, $\hat{BOC} = 1.6745$

two correct calculations for arc lengths

eg $AB = 7 \times 2 \times 1.058 (= 14.8135)$, $7 \times 1.6745 (= 11.7216)$ *(A1)(A1)*

adding **their** arc lengths (seen anywhere)

eg $r\hat{AOB} + r\hat{BOC}$, $14.8135 + 11.7216$, $7(2.116 + 1.6745)$ ***M1***

arc $ABC = 26.5$ (cm) ***A1*** ***N4***

Note: Candidates may work with other interior triangles using a similar method. Check calculations carefully and award marks in line with markscheme.

[6 marks]

Total [14 marks]

9. (a) appropriate approach ***(M1)***

eg $\begin{pmatrix}11\\8\\2\end{pmatrix}+s\begin{pmatrix}4\\3\\-1\end{pmatrix}=\begin{pmatrix}1\\1\\-7\end{pmatrix}+t\begin{pmatrix}2\\1\\11\end{pmatrix}$, $L_1 = L_2$

any **two** correct equations ***A1A1***

eg $11+4s=1+2t$, $8+3s=1+t$, $2-s=-7+11t$

attempt to solve system of equations ***(M1)***

eg $10+4s=2(7+3s)$, $\begin{cases}4s-2t=-10\\3s-t=-7\end{cases}$

one correct parameter ***A1***

eg $s=-2$, $t=1$

$\text{P}(3,\ 2,\ 4)$ (accept position vector) ***A1*** ***N3***

[6 marks]

(b) choosing correct direction vectors for L_1 and L_2 ***(A1)(A1)***

eg $\begin{pmatrix}4\\3\\-1\end{pmatrix}$, $\begin{pmatrix}2\\1\\11\end{pmatrix}$ (or any scalar multiple)

evidence of scalar product (with any vectors) ***(M1)***

eg $a\bullet b$, $\begin{pmatrix}4\\3\\-1\end{pmatrix}\bullet\begin{pmatrix}2\\1\\11\end{pmatrix}$

correct substitution ***A1***

eg $4(2)+3(1)+(-1)(11)$, $8+3-11$

calculating $a\bullet b=0$ ***A1***

Note: Do not award the final ***A1*** without evidence of calculation.

vectors are perpendicular ***AG*** ***N0***

[5 marks]

continued …

Question 9 continued

(c)

> **Note:** Candidates may take different approaches, which do not necessarily involve vectors. In particular, most of the working could be done on a diagram. Award marks in line with the markscheme.

METHOD 1

attempt to find $\overrightarrow{QP}$ or $\overrightarrow{PQ}$ ***(M1)***

correct working (may be seen on diagram) ***A1***

eg $\overrightarrow{QP} = \begin{pmatrix} -4 \\ -3 \\ 1 \end{pmatrix}$, $\overrightarrow{PQ} = \begin{pmatrix} 7 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$

recognizing R is on L_1 (seen anywhere) ***(R1)***

eg on diagram

Q and R are equidistant from P (seen anywhere) ***(R1)***

eg $\overrightarrow{QP} = \overrightarrow{PR}$, marked on diagram

correct working ***(A1)***

eg $\begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 7 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$, $\begin{pmatrix} -4 \\ -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$

R $(-1, -1, 5)$ (accept position vector) ***A1*** ***N3***

METHOD 2

recognizing R is on L_1 (seen anywhere) ***(R1)***

eg on diagram

Q and R are equidistant from P (seen anywhere) ***(R1)***

eg P midpoint of QR, marked on diagram

valid approach to find **one** coordinate of mid-point ***(M1)***

eg $x_P = \frac{x_Q + x_R}{2}$, $2y_P = y_Q + y_R$, $\frac{1}{2}(z_Q + z_R)$

one correct substitution ***A1***

eg $x_R = 3 + (3 - 7)$, $2 = \frac{5 + y_R}{2}$, $4 = \frac{1}{2}(z + 3)$

correct working for one coordinate ***(A1)***

eg $x_R = 3 - 4$, $4 - 5 = y_R$, $8 = (z + 3)$

R $(-1, -1, 5)$ (accept position vector) ***A1*** ***N3***

[6 marks]

Total [17 marks]

10. (a) appropriate approach *(M1)*
eg $\text{P}(R \cap B) + \text{P}(R' \cap B)$, tree diagram,

one correct multiplication *(A1)*
eg 0.2×0.5, 0.24

correct working *(A1)*
eg $0.2 \times 0.5 + 0.8 \times 0.3$, $0.1 + 0.24$

$\text{P}(\text{bus}) = 0.34$ (exact) ***A1*** ***N3***
[4 marks]

(b) recognizing conditional probability *(R1)*
eg $\text{P}(A|B) = \dfrac{\text{P}(A \cap B)}{\text{P}(B)}$

correct working ***A1***
eg $\dfrac{0.2 \times 0.5}{0.34}$

$\text{P}(R|B) = \dfrac{5}{17}$, 0.294 ***A1*** ***N2***
[3 marks]

(c) recognizing binomial probability *(R1)*
eg $X \sim \text{B}(n,\ p)$, $\binom{5}{3}(0.34)^3$, $(0.34)^3(1-0.34)^2$

$\text{P}(X = 3) = 0.171$ ***A1*** ***N2***
[2 marks]

(d) **METHOD 1**
evidence of using complement (seen anywhere) *(M1)*
eg $1 - \text{P}(\text{none})$, $1 - 0.95$

valid approach *(M1)*
eg $1 - \text{P}(\text{none}) > 0.95$, $\text{P}(\text{none}) < 0.05$, $1 - \text{P}(\text{none}) = 0.95$

correct inequality (accept equation) ***A1***
eg $1 - (0.66)^n > 0.95$, $(0.66)^n = 0.05$

$n > 7.209$ (accept $n = 7.209$) *(A1)*

$n = 8$ ***A1*** ***N3***

METHOD 2
valid approach using guess and check/trial and error *(M1)*
eg finding $\text{P}(X \geq 1)$ for various values of n

seeing the "cross over" values for the probabilities ***A1A1***
$n = 7, \text{P}(X \geq 1) = 0.9454$, $n = 8, \text{P}(X \geq 1) = 0.9639$

recognising $0.9639 > 0.95$ *(R1)*

$n = 8$ ***A1*** ***N3***
[5 marks]
Total [14 marks]